Section:

To Understand Complex Statements, Build them from Simple Pieces

We combine statements using words like and, or, not, if and so on.

It was a dark and cloudy night

There are two basic claims/parts

D: "It was dark"

C: "It was cloudy"

We write $C \wedge D$ to denote "C and D"

Truth table:

The conjunction $(C \wedge D)$ is True only when

It is not warm

One claim is rejected.

P: "it is warm"

The negation $\neg P$ asserts that "P is *not* true."

$$\begin{array}{c|c}
P & \neg P \\
\hline
T & \mathbf{F} \\
F & \mathbf{T}
\end{array}$$

The negation $\neg P$ is True only when P is False. Name: _

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This number is either a power of 7 or it is prime

There are two basic parts:

S: This number is a power of 7

P: This number is prime

We write $S \vee P$ to denote "S or P"

S	P	$S \vee P$
Т	T	T
Τ	F	T
F	Т	T
F	F	F

The disjunction $(S \vee P)$ is True when

This definition of "or" is called the **inclusive or**. It is always the meaning of "or" in mathematics.

Warning: translation is subtle. I sometimes write "either P or Q" to mean $P \vee Q$.

Do you want fries xor mashed potatoes with that?

- 1. In mathematics, we always say "or" to mean "inclusive or"
- 2. Many times in *English*, we want to say that you can chose either one option or another option, but not both.
- 3. This operation is called the **exclusive or**.
- 4. In mathematics and programming, we denote exclusive or as P xor Qto emphasize that this is *not* the same thing as $P \vee Q$.

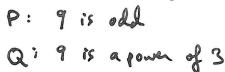
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Worksheet: We Learn Best by Doing!

- 1. For following statement, (1) identify its basic parts/components and label them with letters, and (2) translate the sentence symbolically (e.g. using $P \wedge Q$, $P \vee Q$, $\neg P$).
 - (a) "The number 9 is both odd and a power of 3."





(b) "At least one of the numbers x and y are positive"

P: 200000 X >0



(c) "I am unhappy"



2. Consider the statement "It is a challenging but enjoyable problem."

Determine if the statement is true in each of the following worlds.

• A challenging, enjoyable problem.

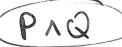
• A challenging, unplesant problem.

False: Not enjoyable

• An easy, enjoyable problem.

• An easy, unpleasant problem.

Based on your answers above, how should you use the standard logical connectives to translate expressions of the form "P but Q"?



3. Write a truth table for the sentence $\neg (P \land Q) \lor (\neg P)$

P	QA	78	PAQ	¬(PAQ)	7(PAQ) V (7P)
	T	F	T	F	F
7		E	F	T	T
F	T	T	F	T	T
F	F	+	F	T	T

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4. Write a truth table for the sentence $P \wedge (Q \vee R)$

	P	Q	R	QVR	PA(QVR)
	T	T	T	T	T
	T	1	F	1	T
	T	F	T	T	T
_	T	F	F	F	F
	F	T	T	T	(F)
	F	T	F	T	F
	9	F	T	T	F
	F	F	F	F	F
	}	ı			V

NOTE:
must list rows
in Standard
order

5. Write a truth table for the sentence $(P \wedge Q) \vee R$

PIQRI	PAQ	(PAQ) VR	
TIT	T	T	
TTFT	F	T	*************
TFF	F	F	
FIT	F		
FFT	 	F	-
FFF	F	E	
' '			

Do the sentences $P \wedge (Q \vee R)$ and $(P \wedge Q) \vee R$ mean the same thing? Justify your answer using *both* the truth tables above *and* a common sense explanation.

	They do to rear the same tour	
0	different truth value in rows 587]	Even happy-
②	Let P: "happy Q: I'm wealthy, R: I'm west. D is I'm unhappy but wealthy &wise, PAQVR) Whereas (PAQ) VR	is false is true
	4 Const I'm	WILE

6. Consider the statement "I'm not happy but I'm not unhappy." Identify its most basic parts/components and translate the sentence into logical symbols. (You must define each letter you use.)

let P: I am happy the sentence states 7H 1 7 (7H)

7. Translate the sentence $((P \land Q) \lor (\neg P \land \neg Q)) \land \neg (\neg Q \land R)$ into English, where the propositions P, Q, R denote the following

: "It is warm"

"I go swimming"

"I am happy"

Note: You must use English punctuation to ensure that your English statement has the correct meaning (to preserve the desired order of operations).

You may use commas or even multiple sentences, but you may not use parentheses.

is worn and I go swimming, it is not worn and I do not go swimming. in either case, it will rever be true that I & am happy when I did not go swimming.

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Important Logical Equivalences

You should memorize each of the following logical equivalences.

You should also make sure you understand why it is true. Practice using both (1) "common sense reasoning" and also (2) giving a careful argument using truth tables and the definition of logical equivalence.

$$\neg (P \land Q) \equiv \neg P \lor \neg Q$$

De Morgan's Laws

$$\neg(P \lor Q) \equiv \neg P \land \neg Q$$

$$P \wedge Q \equiv Q \wedge P$$

Commutative Laws

$$P\vee Q\equiv Q\vee P$$

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$$

Distributive Laws

$$P\vee (Q\wedge R)\equiv (P\vee Q)\wedge (P\vee R)$$

$$P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$$

Associative Laws

$$P \lor (Q \lor R) \equiv (P \lor Q) \lor R$$

Note: the distributive law $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$ has the same form as the algebra rule $a \cdot (b+c) = a \cdot b + a \cdot c$.

Is there a connection between logic and algebra?

Most Sentences are Not Equivalent

Be suspicious of any half-remembered logical equivalence. Your intuition can decieve you!

You need to know the main equivalences, so that you can avoid making common errors!