Name: Solutions Section: ______________

To Understand Complex Statements, Build them from Simple Pieces

We combine statements using words like *and, or, not, if* and so on.

It was a dark and cloudy night

There are two basic claims/parts

D : "It was dark"

C : "It was cloudy"

We write $C \wedge D$ to denote "C *and* D"

Truth table:

C	D	$C \wedge D$
T	T	T
T	F	F
F	T	F
F	F	F

The *conjunction* $(C \wedge D)$ is True *only* when both parts are true.

It is not warm

One claim is rejected.

P : "it is warm"

The negation $\neg P$ asserts that "P is *not* true."

P	$\neg P$
T	F
F	T

The negation $\neg P$ is True *only* when P is False.

Name: ____________________ Section: ____________________

This number is either a power of 7 or it is prime

There are two basic parts:

S : This number is a power of 7

P : This number is prime

We write $S \vee P$ to denote "S *or* P"

S	P	$S \vee P$
T	T	T
T	F	T
F	T	T
F	F	F

The *disjunction* $(S \vee P)$ is True when at least one part is true.

This definition of "or" is called the **inclusive or**. It is always the meaning of "or" in mathematics.

Warning: translation is subtle. *I* sometimes write "either P or Q" to mean $P \vee Q$.

Do you want fries *xor* mashed potatoes with that?

1. In mathematics, we *always* say "or" to mean "inclusive or"
2. Many times in *English*, we want to say that
   you can chose either one option or another option, *but not both.*
3. This operation is called the **exclusive or**.
4. In mathematics and programming, we denote exclusive or as P *xor* Q
   to emphasize that this is *not* the same thing as $P \vee Q$.

Name: ______________________ Section: ______________________

Worksheet: We Learn Best by Doing!

1. For following statement, (1) identify its basic parts/components and label them with letters, and (2) translate the sentence symbolically (e.g. using $P \wedge Q$, $P \vee Q$, $\neg P$).

   (a) "The number 9 is both odd and a power of 3."

   P: 9 is odd

   Q: 9 is a power of 3

   $P \wedge Q$

   (b) "At least one of the numbers x and y are positive"

   P: $x > 0$

   Q: $y > 0$

   $P \vee Q$

   (c) "I am unhappy"

   H: I am happy

   $\neg H$

2. Consider the statement "It is a challenging but enjoyable problem."

   Determine if the statement is true in each of the following worlds.

   - A challenging, enjoyable problem. true
   - A challenging, unplesant problem. False: Not enjoyable
   - An easy, enjoyable problem. False: Not challenging
   - An easy, unpleasant problem. False : neither!

   Based on your answers above, how should you use the standard logical connectives to translate expressions of the form "P but Q"?

   $P \wedge Q$

3. Write a truth table for the sentence $\neg(P \wedge Q) \vee (\neg P)$

P	Q	$\neg P$	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg(P \wedge Q) \vee (\neg P)$
T	T	F	T	F	F
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	F	T	T

Name: ______________________ Section: ______________________

4. Write a truth table for the sentence $P \wedge (Q \vee R)$

P	Q	R	$Q \vee R$	$P \wedge (Q \vee R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

NOTE: **must** list rows in standard order

5. Write a truth table for the sentence $(P \wedge Q) \vee R$

P	Q	R	$P \wedge Q$	$(P \wedge Q) \vee R$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

Do the sentences $P \wedge (Q \vee R)$ and $(P \wedge Q) \vee R$ mean the same thing? Justify your answer using *both* the truth tables above *and* a common sense explanation.

They do NOT mean the same thing

① different truth value in rows 5 & 7

② let P: I'm happy, Q: I'm wealthy, R: I'm wise.

if I'm unhappy but wealthy & wise, $P \wedge (Q \vee R)$ is false. (I'm not even happy)
whereas $(P \wedge Q) \vee R$ is true (at least I'm wise)

Name: ____________________ Section: ____________________

6. Consider the statement "I'm not happy but I'm not unhappy."
Identify its most basic parts/components and translate the sentence into logical symbols. (You must define each letter you use.)

let P: I am happy

the sentense states $\neg H \wedge \neg(\neg H)$

7. Translate the sentence $((P \wedge Q) \vee (\neg P \wedge \neg Q)) \wedge \neg(\neg Q \wedge R)$ into English, where the propositions P, Q, R denote the following

P	:	"It is warm"
Q	:	"I go swimming"
R	:	"I am happy"

Note: You *must* use English punctuation to ensure that your English statement has the correct meaning (to preserve the desired order of operations).

You may use commas or even multiple sentences, but you may *not* use parentheses.

It is warm and I go swimming,
or it is not warm and I do not go swimming.
(and)
But in either case,
it will never be true that I am happy
when I did not go swimming.

Name: ______________________________ Section: ______________________________

Important Logical Equivalences

You should **memorize** each of the following logical equivalences.

You should also make sure you understand *why* it is true. Practice using both (1) "common sense reasoning" and also (2) giving a careful argument using truth tables and the definition of logical equivalence.

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$
$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

De Morgan's Laws

$$P \wedge Q \equiv Q \wedge P$$
$$P \vee Q \equiv Q \vee P$$

Commutative Laws

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$$
$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$

Distributive Laws

$$P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$$
$$P \vee (Q \vee R) \equiv (P \vee Q) \vee R$$

Associative Laws

Note: the distributive law $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$ has the same form as the algebra rule $a \cdot (b + c) = a \cdot b + a \cdot c$.

Is there a connection between logic and algebra?

Most Sentences are Not Equivalent

Be suspicious of any half-remembered logical equivalence. Your intuition can decieve you!

You need to know the main equivalences, so that you can *avoid making common errors!*